GURU NANAK ACADEMY SAFIDON

Class XI

Maths

Time: 3 hours M.M. – 100

(i) All the questions are compulsory

(ii) The Question paper consists of 29 questions.

(iii) Questions 1 – 4 in Section A are very short answer type questions carrying 1 mark each.

(iv) Questions 5 – 12 in Section B are short answer type questions carrying 2 marks each.

(v) Questions 13 – 23 in Section C are long answer – I type questions carrying 4 marks each.

(vi) Questions 24 – 29 in Section D are long answer type – II questions carrying 6 marks each.

Section – A

1. Find the centre and radius of the circle
$x^2 + y^2 - 8y = 10y - 12 = 0$

2. Find the value of n such that $n_{P_s} = 42\ n_{P_3}\ n > 4$

3. Solve $\sqrt{5}x^2$ + x + $\sqrt{5}$ = 0

4. Identify the type of "Or" used in the following statements and check whether the statements are true or false

(i) $\sqrt{2}$ is a rational number or an irrational number.

(ii) To enter into a public library children need an identity card from the school or a letter from the school aurhorities.

Section – B

5. Draw appropriate Venn diagram for each of following:

a. $(A \cup B)^I$ c. $(A - B) \cup (B - A)$

b. $(A \cap B)^I$ d. A^I

6. Prove that $\sum_{r=0}^{n} 3^r\ \ {}^nc_r\ = 4^n$

7. Find the coordinates of the foci, the vertices, the length of major and minor axis of the ellipse
$9x^2 + 4y^2 = 36$

8. Show that the points P(-2, 3, 5), Q(1, 2, 3) and R(7, 0, -1) are collinear.

9. Evaluate:

a. $\lim_{x\to 1}\left[\frac{x^2+1}{x+100}\right]$

b. $\lim_{x\to 1}\left[\frac{x-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right]$

10. Show that the statement
A: "If x is a real number such that $x^3+4x=0$,then x is o" is true by
(a) Method of Contraction
(b) Method of Contrapositive

11. A committee of two persons is selected from two men and two women. What is the probability that committee will have
(a) no man? (b) one man?

12. If E and F are events such that P(E) = $\frac{1}{4}$, P(F) = $\frac{1}{2}$ and P(E and F) = $\frac{1}{8}$, Find (a) P(E OR F) (b) P (not E and not F)

Section – C

13. In a survey of 60 people it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find:
(a) The number of people who read at least one of the newspapers.
(b) The number of people who read exactly one newspaper.

14. Let $A=\{1,2,3\}, B=\{3,4\}\ and\ C=\{4,5,6\}$. Find
(a) $A\times(B\cap C)$ (c) $A\times(B\cup C)$
(b) $(A\times B)\cap(A\times C)$ (d) $(A\times B)\cup(A\times C)$

15. Prove that $\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}=\cot 3x$.

16. Prove that 1.2.3 + 2.3.4 + _____ + n. (n+1) . (n+2) = $\frac{n(n+1)(n+2)(n+3)}{4}$ for all n∈ N.

17. Convert Z= $\frac{1+7i}{(2-i)^2}$ in the polar form.

18. Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements,
a. do the words start with P.
b. do all the vowels always occur together.
c. do the vowels never occur together.
d. do the words begin with I and end in P?

19. Find a, if the 17th and 18th terms in the expansion of $(2+a)^{50}$ are equal.

20. If S_1, S_2 and S_3 are the sums of first n natural numbers, their square, their cubes respectively, show that $9S_2^2=S_3(1+8S_1)$.

21. Find the equations of all the meridians of the triangle ABC whose vertices are A(2, 5), B(-4, 9) and C(-2, -1)

22. Find the equation of the circle passing through the points (2,3) and (-1, 1) and whose centre is on the line x-3y-11=0

OR

An arc is in the form of a parabola with its axis vertical. The arc is 10 m high and 5 m wide at the base. How wide is it 2 m from the vertex of the parabola.

23. Find the ratio in which the line joining the points (1, 2, 3) and (-3, 4, -5) is divided by the xy - plane. Also, find the coordinates of the point of division.

OR

Prove that the line joining the vertices of a tetrahedron to the centroids of the opposite faces are concurrent.

Section – D

24. Prove that

(a) $\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)} = \tan 6x$

(b) $(1 + \cot x - \operatorname{cosec} x)\ 1 + \tan x + \sec x) = 2$

25. Exhibit graphically the solution set of the linear in equations

$x + y \leq 5$

$4x + y \geq 4$

$x + 5y \geq 5$

$x \leq 4$

$x \leq 3$

26. The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

OR

The sum of two numbers is 6 times their geometric mean, show that the numbers are in the ratio of $(3+2\sqrt{2}):(3-2\sqrt{2})$.

27. Differentiate the following functions with respect to x.

(a) $x^2 \sin x + \frac{1}{x^2}$

(b) $\frac{ax^2+bx+c}{\sqrt{x}}$

(c) $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

OR

Evaluate:

$$\lim_{x\to 0} \frac{8}{x^8}\left[1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{4}+\cos\frac{x^2}{4}\right]$$

28. Calculate the mean and standard deviation for the following table given the age distribution of a group of people:

29. (a) The owner of a milk store finds that he can sell 980 litres of milk each week at Rs. 14/litre and 1220 litres of milk each week at Rs 16/litre. Assumung a linear relationship between selling price and demand, how many litres could he sell weekly at Rs 17/litre?

(c) Find the angle between the lines $y-\sqrt{3x}-5=0\ and\ \sqrt{3}\,y-x+6=0$.